

Exact Mapping of the 2+1 Dirac Oscillator onto the Jaynes-Cummings Model: Ion-Trap Experimental Proposal

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We study the dynamics of the 2+1 Dirac oscillator exactly and find spin oscillations due to a *Zitterbewegung* of purely relativistic origin. We find an exact mapping of this quantum-relativistic system onto a Jaynes-Cummings model, describing the interaction of a two-level atom with a quantized single-mode field. This equivalence allows us to map a series of quantum optical phenomena onto the relativistic oscillator, and viceversa. We make a realistic experimental proposal, at reach with current technology, for studying the equivalence of both models using a single trapped ion.

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Current technology has allowed the implementation of the paradigmatic nonrelativistic quantum harmonic oscillator in a single trapped ion [1], one of the most fundamental toy models in any quantum mechanical textbook. However, its relativistic version, the so-called Dirac oscillator [2, 3], remains still far from any possible experimental consideration for different fundamental and technical reasons. We will show here that available experimental tools may allow the implementation of the relativistic Dirac oscillator in a single nonrelativistic trapped ion.

The Dirac oscillator was introduced as an instance of a relativistic wave equation such that its nonrelativistic limit leads to the well-known Schrödinger equation for the harmonic oscillator. This is achieved by introducing the following coupling in the Dirac equation

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \left[\sum_{j=1}^3 c\alpha_j (p^j - im\beta\omega r^j) + \beta mc^2 \right] |\Psi\rangle, \quad (1)$$

where $|\Psi\rangle$ is the Dirac 4-component bispinor corresponding to a quantum relativistic spin- $\frac{1}{2}$ particle, like the electron, c is the speed of light, m is the particle rest mass, and α_j , β , are the Dirac matrices in the standard representation. The interacting Hamiltonian is linear in both momentum p^j and position r^j , $j = x, y, z$, and ω turns out to be the harmonic oscillator frequency. Remark that when $\omega = 0$ we recover the standard Dirac equation [4]. The Dirac oscillator looks like a particular gauge transformation $\mathbf{p} \rightarrow \mathbf{p} - \frac{e}{c}\mathbf{A}$ that is linear in position, but the presence of the i and the β matrix makes a crucial difference. Demanding the correct energy-momentum relation for a relativistic free particle $E = \sqrt{p^2c^2 + m^2c^4}$, these matrices are 4×4 dimensional and must obey a Clifford algebra given by the anticommutation relations

$$\begin{aligned} \alpha_j \alpha_k + \alpha_k \alpha_j &= 2\delta_{jk}, \\ \alpha_j \beta + \beta \alpha_j &= 0. \end{aligned} \quad (2)$$

There has been a growing interest in simulating quantum relativistic effects in other physical systems, such as

black hole evaporation in Bose-Einstein condensates [5] and the Unruh effect in an ion chain [6]. Another astonishing relativistic prediction is the *Zitterbewegung* [4], a helicoidal motion realized by the average position of a relativistic fermion, which has been discussed in the context of condensed matter systems [7] and the free-particle Dirac equation in a single ion [8].

Here, we shall be concerned with the Dirac oscillator model in 2+1 dimensions, since it is in this setting where we can establish a precise equivalence with the Jaynes-Cummings (JC) model [9]. In two spatial dimensions, the solution to the Clifford algebra (2) is given by the 2×2 Pauli matrices: $\alpha_x = \sigma_x, \alpha_y = \sigma_y, \beta = \sigma_z$. In this case, $|\Psi\rangle$ can be described by a 2-component spinor which mixes spin up and down components with positive and negative energies. In particular, the Dirac oscillator model now takes the form

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \left[\sum_{j=1}^2 c\sigma_j (p^j - im\sigma_z\omega r^j) + \sigma_z mc^2 \right] |\Psi\rangle. \quad (3)$$

In this paper, we shall provide the complete (eigenstates and energies) and exact solution of the 2D Dirac oscillator in order to study its relativistic dynamics, where certain collapses and revivals in the spin degree of freedom appear as a consequence of *Zitterbewegung*. In addition, we derive an exact mapping of the 2+1 Dirac oscillator onto the JC model, an archetypical quantum optical system. Furthermore, we propose the simulation of this relativistic dynamics in a single trapped ion, a physical setup possessing outstanding coherence features.

Considering the spinor $|\Psi\rangle := [|\psi_1\rangle, |\psi_2\rangle]^t$, equation (3) becomes a set of coupled equations

$$\begin{aligned} (E - mc^2)|\psi_1\rangle &= c[(p_x + im\omega x) - i(p_y + im\omega y)]|\psi_2\rangle, \\ (E + mc^2)|\psi_2\rangle &= c[(p_x - im\omega x) + i(p_y - im\omega y)]|\psi_1\rangle. \end{aligned} \quad (4)$$

In order to find the solutions, it is convenient to introduce

the following chiral creation and annihilation operators

$$\begin{aligned} a_r &:= \frac{1}{\sqrt{2}}(a_x - ia_y), & a_r^\dagger &:= \frac{1}{\sqrt{2}}(a_x^\dagger + ia_y^\dagger), \\ a_l &:= \frac{1}{\sqrt{2}}(a_x + ia_y), & a_l^\dagger &:= \frac{1}{\sqrt{2}}(a_x^\dagger - ia_y^\dagger), \end{aligned} \quad (5)$$

where $a_x, a_x^\dagger, a_y, a_y^\dagger$ are the usual annihilation and creation operators of the harmonic oscillator $a_i^\dagger = \frac{1}{\sqrt{2}}(\frac{1}{\Delta}r^i - i\frac{\Delta}{\hbar}p^i)$, and $\Delta = \sqrt{\hbar/m\omega}$ represents the ground state oscillator width. The orbital angular momentum may also be expressed as

$$L_z = \hbar(a_r^\dagger a_r - a_l^\dagger a_l), \quad (6)$$

which leads to a physical interpretation of a_r^\dagger and a_l^\dagger . These operators create a right or left quantum of angular momentum, respectively, and are known hence as circular creation-annihilation operators. Equations (4) can be rewritten in the language of these circular operators

$$\begin{aligned} |\psi_1\rangle &= i\frac{2mc^2\sqrt{\xi}}{E-mc^2} a_l^\dagger |\psi_2\rangle, \\ |\psi_2\rangle &= -i\frac{2mc^2\sqrt{\xi}}{E+mc^2} a_l |\psi_1\rangle, \end{aligned} \quad (7)$$

where $\xi := \hbar\omega/mc^2$ controls the nonrelativistic limit. In order to find the energy spectrum we shall solve the associated Klein-Gordon equation, which can be derived from Eqs. (7) as follows

$$\begin{aligned} (E^2 - m^2c^4)|\psi_1\rangle &= 4m^2c^4\xi a_l^\dagger a_l |\psi_1\rangle, \\ (E^2 - m^2c^4)|\psi_2\rangle &= 4m^2c^4\xi (1 + a_l^\dagger a_l)|\psi_2\rangle. \end{aligned} \quad (8)$$

These equations can be simultaneously diagonalized writing the spinor in terms of the left chiral quanta basis

$$|n_l\rangle = \frac{1}{\sqrt{n_l!}} (a_l^\dagger)^{n_l} |\text{vac}\rangle, \quad (9)$$

where $n_l = 0, 1, \dots$. The energies can be expressed as

$$\begin{aligned} (E_{n_l}^2 - m^2c^4)|n_l\rangle &= 4m^2c^4\xi n_l |n_l\rangle, \\ (E_{n_l'}^2 - m^2c^4)|n_l'\rangle &= 4m^2c^4\xi (1 + n_l')|n_l'\rangle. \end{aligned} \quad (10)$$

Since both components $|\psi_1\rangle$ and $|\psi_2\rangle$ belong to the same solution, the energies must be the same $E_{n_l'} = E_{n_l}$. This physical requirement sets up a constraint on the quantum numbers $n_l =: n_l' + 1$. Note that, following (6), the state $|n_l\rangle$ corresponds to a negative angular momentum. The energy spectrum can be described as follows

$$E = \pm E_{n_l} = \pm mc^2 \sqrt{1 + 4\xi n_l}. \quad (11)$$

To find the corresponding eigenstates, we go back to Eq. (7), and after normalization we arrive at the expression for the positive and negative energy eigenstates

$$|\pm E_{n_l}\rangle = \begin{bmatrix} \sqrt{\frac{E_{n_l} \pm mc^2}{2E_{n_l}}} |n_l\rangle \\ \mp i \sqrt{\frac{E_{n_l} \mp mc^2}{2E_{n_l}}} |n_l - 1\rangle \end{bmatrix}, \quad (12)$$

where the quantum number is now restricted to $n_l = 1, 2, \dots$. In this way, we have solved the two-dimensional Dirac oscillator describing the energy spectrum and the eigenstates in terms of circular quanta. The distinction between Dirac and Klein-Gordon eigenstates is an important point in order to understand the dynamics of the 2+1 Dirac oscillator and its realization in an ion trap.

The eigenstates of the 2D Dirac oscillator can be expressed more transparently in terms of 2-component Pauli spinors $|\chi_\uparrow\rangle$ and $|\chi_\downarrow\rangle$

$$\begin{aligned} | + E_{n_l} \rangle &= \alpha_{n_l} |n_l\rangle |\chi_\uparrow\rangle - i\beta_{n_l} |n_l - 1\rangle |\chi_\downarrow\rangle, \\ | - E_{n_l} \rangle &= \beta_{n_l} |n_l\rangle |\chi_\uparrow\rangle + i\alpha_{n_l} |n_l - 1\rangle |\chi_\downarrow\rangle, \end{aligned} \quad (13)$$

where $\alpha_{n_l} := \sqrt{\frac{E_{n_l} + mc^2}{2E_{n_l}}}$ and $\beta_{n_l} := \sqrt{\frac{E_{n_l} - mc^2}{2E_{n_l}}}$ are real.

From this expression we observe that the energy eigenstates present entanglement between the orbital and spin degrees of freedom. This property is extremely important since the following initial state

$$|\Psi(0)\rangle := |n_l - 1\rangle |\chi_\downarrow\rangle = i\beta_{n_l} | + E_{n_l} \rangle - i\alpha_{n_l} | - E_{n_l} \rangle \quad (14)$$

superposes states with positive and negative energies, and this is the fundamental ingredient that leads to *Zitterbewegung* in relativistic quantum dynamics. This phenomenon, due to the interference of positive and negative energies, has never been observed experimentally. The reason is that the amplitude of these rapid oscillations lies below the Compton wavelength, where pair creation is allowed, and the one-particle interpretation falls down.

Now, the evolution of this initial state can be expressed in the energy basis as

$$|\Psi(t)\rangle = i\beta_{n_l} | + E_{n_l} \rangle e^{-i\omega_{n_l}t} - i\alpha_{n_l} | - E_{n_l} \rangle e^{i\omega_{n_l}t}, \quad (15)$$

where

$$\omega_{n_l} := \frac{E_{n_l}}{\hbar} = \frac{mc^2}{\hbar} \sqrt{1 + 4\xi n_l} \quad (16)$$

describes the frequency of oscillations. Writing this evolved state in the language of Pauli spinors,

$$\begin{aligned} |\Psi(t)\rangle &= \left(\cos \omega_{n_l}t + \frac{i}{\sqrt{1 + 4\xi n_l}} \sin \omega_{n_l}t \right) |n_l - 1\rangle |\chi_\downarrow\rangle \\ &+ \left(\sqrt{\frac{4\xi n_l}{1 + 4\xi n_l}} \sin \omega_{n_l}t \right) |n_l\rangle |\chi_\uparrow\rangle, \end{aligned} \quad (17)$$

we observe an oscillatory dynamics between $|n_l\rangle |\chi_\uparrow\rangle$ and $|n_l - 1\rangle |\chi_\downarrow\rangle$. The initial state, $|n_l - 1\rangle |\chi_\downarrow\rangle$, which has spin-down and $n_l - 1$ quanta of left orbital angular momentum, evolves exchanging a quantum of angular momentum from the spin to the orbital motion.

The dynamics described in (17) is completely similar to the atomic Rabi oscillations occurring in the Jaynes-Cummings model, though arising from a completely different reason. Whereas the Rabi oscillations in the

Jaynes-Cummings model are caused by the interaction of a quantized electromagnetic field with a two-level atom, the relativistic oscillations are caused by the interference of positive and negative energy states and therefore constitute a clear signature of *Zitterbewegung* [4].

To clarify this issue further, we calculate the time evolution of the following physical observables, that catch the full essence of the system dynamics,

$$\begin{aligned}\langle L_z \rangle_t &= -(n_l - 1)\hbar - \frac{4\xi n_l}{1+4\xi n_l} \hbar \sin^2 \omega_{n_l} t, \\ \langle S_z \rangle_t &= -\frac{\hbar}{2} + \frac{4\xi n_l}{1+4\xi n_l} \hbar \sin^2 \omega_{n_l} t, \\ \langle J_z \rangle_t &= \hbar(\frac{1}{2} - n_l),\end{aligned}\quad (18)$$

where $J_z = L_z + S_z$ stands for the z -component of the total angular momentum. The latter relations describe a certain oscillation in the spin and orbital angular momentum, while the total angular momentum is conserved due to the existent invariance under rotations around the z -axis. It is important to highlight that these oscillations have a pure relativistic nature. In the nonrelativistic limit $\xi \ll 1$, these oscillations become vanishingly small

$$\begin{aligned}\langle L_z \rangle_t &= -(n_l - 1)\hbar - 4\xi n_l \hbar \sin^2 \Omega_{n_l} t + \mathcal{O}(\xi^2), \\ \langle S_z \rangle_t &= -\frac{\hbar}{2} + 4\xi n_l \hbar \sin^2 \Omega_{n_l} t + \mathcal{O}(\xi^2),\end{aligned}\quad (19)$$

where $\Omega_{n_l} := mc^2(1 + 2\xi n_l)/\hbar$ stands for the oscillation frequency in the nonrelativistic limit. In this limit the negative energy components are negligible and therefore the *Zitterbewegung* disappears.

The results discussed so far allow a precise mapping between two seemingly unrelated models: the Jaynes-Cummings model of Quantum Optics and the 2D Dirac oscillator. Starting from Eq. (7), we may write the Dirac oscillator Hamiltonian as

$$\begin{aligned}H &= 2imc^2\sqrt{\xi} \left(a_l^\dagger |\psi_2\rangle\langle\psi_1| - a_l |\psi_1\rangle\langle\psi_2| \right) + mc^2\sigma_z \\ &= \hbar(g\sigma^- a_l^\dagger + g^*\sigma^+ a_l) + mc^2\sigma_z,\end{aligned}\quad (20)$$

where σ^+ , σ^- , are the spin raising and lowering operators, and $g := 2imc^2\sqrt{\xi}/\hbar$ is the coupling strength between orbital and spin degrees of freedom. In Quantum Optics, this Hamiltonian describes a Jaynes-Cummings interaction, that has been studied in cavity QED and trapped ions [1, 10], among others. Within this novel perspective, the electron spin can be associated with a two-level atom, and the orbital circular quanta with the ion quanta of vibration, i.e., phonons. As we will see below, the central result of Eq. (20) allows both physical systems, the JC model and the 2D Dirac oscillator, to exchange a wide range of important applications.

We will show now how to implement the dynamics of Eq. (3) in a single ion inside a Paul trap, which was shown to follow the dynamics of Eq. (20). The Dirac spinor will be described by two metastable internal states, $|g\rangle$ and $|e\rangle$, as follows

$$|\Psi\rangle := |\psi_1\rangle|e\rangle + |\psi_2\rangle|g\rangle\quad (21)$$

while the circular angular momentum modes will be represented by two ionic vibrational modes, a_x and a_y . Current technology allows an overwhelming coherent control of ionic internal and external degrees of freedom [1]. There, three paradigmatic interactions, the carrier, red-, and blue-sideband excitations, can be implemented at will, independently or simultaneously [11]. For example, using appropriately tuned lasers, it is possible to produce the following interactions

$$\begin{aligned}H_i^{\text{JC}} &= \hbar\eta_i\tilde{\Omega}_i \left[\sigma^+ a_i e^{i\phi} + \sigma^- a_i^\dagger e^{-i\phi} \right] + \hbar\delta_i\sigma_z, \\ H_i^{\text{AJC}} &= \hbar\eta_i\tilde{\Omega}_i \left[\sigma^+ a_i^\dagger e^{i\varphi} + \sigma^- a_i e^{-i\varphi} \right],\end{aligned}\quad (22)$$

where $\{a_i, a_i^\dagger\}$, with $i = x, y$, are the phonon annihilation and creation operators in directions x and y , ν_i are the natural trap frequencies, $\eta_i := k_i\sqrt{\hbar/2M\nu_i}$ are the associated Lamb-Dicke parameters depending on the ion mass M and the wave vector \mathbf{k} , δ_i and $\tilde{\Omega}_i$ are the excitation coupling strengths and ϕ, φ , the red and blue sideband phases. Remark that the term $\hbar\delta_i\sigma_z$, in H_i^{JC} of Eq. (22), stems from a detuned JC excitation.

A suitable combination of the above introduced excitations (22), with proper couplings and relative phases, can reproduce the following Hamiltonian

$$H = c[\sigma_x^{ge} p_x + \sigma_y^{ge} p_y] + m\omega c[\sigma_x^{ge} y - \sigma_y^{ge} x] + mc^2\sigma_z^{ge}\quad (23)$$

with $\sigma_x^{ge} := |g\rangle\langle e| + |g\rangle\langle e|$, $\sigma_y^{ge} := -i(|e\rangle\langle g| - |e\rangle\langle g|)$, $\sigma_z^{ge} := |e\rangle\langle e| - |g\rangle\langle g|$, and the following parameter correspondence

$$\begin{aligned}c &= \sqrt{2}\eta\tilde{\Omega}\tilde{\Delta}, \\ mc^2 &= \hbar\delta, \\ m\omega c &= \hbar\sqrt{2}\eta\tilde{\Omega}\tilde{\Delta}^{-1},\end{aligned}\quad (24)$$

where $\tilde{\Delta} := \tilde{\Delta}_i$ is the width of the motional ground state, $\tilde{\Omega} := \tilde{\Omega}_i$, $\eta := \eta_i$, $\forall i = x, y$. The remarkable equivalence of the Dirac oscillator Hamiltonian (3) and the interaction in Eq. (23) shows that it is possible to reproduce the 2D Dirac oscillator, with all its quantum relativistic effects, in a controllable quantum system as a single trapped ion.

For the sake of illustration, note that the effective terms appearing in Eq. (23) can be achieved by suitable linear combinations of H_i^{JC} and H_i^{AJC} in (22),

$$\begin{aligned}i = x, \delta_x = \delta, \phi = \frac{3\pi}{2}, \varphi = \frac{\pi}{2} &\rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}\sigma_x^{ge} p_x + \hbar\delta\sigma_z^{ge}, \\ i = y, \delta_y = 0, \phi = 0, \varphi = \pi &\rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}\sigma_y^{ge} p_y, \\ i = x, \delta_x = 0, \phi = \frac{\pi}{2}, \varphi = \frac{\pi}{2} &\rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}^{-1}\sigma_y^{ge} x, \\ i = y, \delta_y = 0, \phi = 0, \varphi = 0 &\rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}^{-1}\sigma_x^{ge} y.\end{aligned}\quad (25)$$

Note that in the trapped ion picture, the important parameter $\xi = 2(\eta\tilde{\Omega}/\delta)^2$ can take on all positive values, assuming available experimental parameters: $\eta \sim 0.1$, $\tilde{\Omega} \sim 0 - 10^6 \text{ Hz}$, and $\delta \sim 0 - 10^6 \text{ Hz}$ [1]. The ability to experimentally tune these parameters will allow the experimenter to study otherwise inaccessible physical regimes

that entail relativistic and nonrelativistic phenomena. For example, the *Zitterbewegung* is encoded in the spin degree of freedom, and we can associate Rabi oscillations to the interference of positive and negative energy solutions. Setting the initial state $|0\rangle|\chi_1\rangle \leftrightarrow |0\rangle|g\rangle$, the internal degree of freedom evolves according to Eq. (18)

$$\langle S_z \rangle_t = -\frac{\hbar}{2} + \frac{4\xi}{1+4\xi} \hbar \sin^2 \omega_1 t, \quad (26)$$

where $\omega_1 = \delta\sqrt{1+4\xi}$, see Eq. (16), stands for the frequency of the *Zitterbewegung* oscillations and can take on a wide variety of measurable values.

In order to simulate this dynamics in an ion-trap tabletop experiment, the ion must be cooled down to its vibrational ground state $|0\rangle$, with a current efficiency above 99% [1]. To estimate the observable (26), one can make use of the powerful tool called electron shelving, where

$$\langle S_z \rangle_t = \frac{\hbar}{2} [2P_e(t) - 1] \quad (27)$$

can be obtained through the measurement of the probability of obtaining the ionic excited state $P_e(t)$ with extraordinary precision.

Another fundamental result of the JC model which can be mapped straightforward to the Dirac oscillator is the existence of collapses and revivals in the atomic population, which is claimed to be a direct evidence of the quantization of the electromagnetic field. To produce this effect an initial state $|z\rangle|g\rangle$ is required, where $|z\rangle$ is an initial circular coherent state,

$$|\Psi(0)\rangle = e^{-|z|^2/2} \sum_{n_l=0}^{\infty} \frac{z^{n_l}}{\sqrt{n_l!}} |n_l\rangle|g\rangle, \quad (28)$$

with $z \in \mathbb{C}$. After an interaction time t ,

$$\langle S_z \rangle_t = -\frac{\hbar}{2} + \hbar \sum_{n_l=0}^{\infty} \frac{4\xi(n_l+1)|z|^{2n_l} e^{-|z|^2}}{[1+4\xi(n_l+1)]n_l!} \sin^2(\omega_{n_l+1}t). \quad (29)$$

This expression can be understood as an interference effect of terms with different frequencies ω_{n_l+1} leading to collapses and revivals. A novel feature of the Dirac oscillator is the appearance of these collapses and revivals in the orbital circular motion of the particle, reflected in

$$\langle L_z \rangle_t = -\hbar|z|^2 - \hbar \sum_{n_l=0}^{\infty} \frac{4\xi(n_l+1)|z|^{2n_l} e^{-|z|^2}}{[1+4\xi(n_l+1)]n_l!} \sin^2(\omega_{n_l+1}t). \quad (30)$$

The generation of an initial circular coherent state will require two sequential applications of the technique described in Ref. [1] on an initial motional ground state. These two operations should be applied with a relative phase such that $D_l(z) = D_x(z)D_y(-iz)$, where $D_j(z) = e^{za_j^\dagger - z^*a_j}$, $j = x, y$. The observable of Eq. (29) can be

measured via a similar electron-shelving technique, while the observable of Eq. (30) can be measured via the mapping of the collective motional state onto the internal degree of freedom [1].

It is worth mentioning that the chiral partner of the 2D Dirac oscillator Hamiltonian (3) can be obtained through the substitution $\omega \rightarrow -\omega$, and consists on right-handed quanta. This Hamiltonian presents similar features as those discussed above, and can be exactly mapped onto an anti-Jaynes-Cummings interaction

$$H = \hbar(ga_r\sigma^- + g^*a_r^\dagger\sigma^+) + mc^2\sigma_z, \quad (31)$$

with similar parameters. It is precisely this chirality which allows an exact mapping between the JC, AJC, and the lefthanded and righthanded 2D Dirac oscillator. This essential property, missing in the 3D case, forbids an exact mapping of Eq. (1) onto a JC-like Hamiltonian.

In conclusion, we have demonstrated the exact mapping of the 2+1 Dirac oscillator onto a Jaynes-Cummings model, allowing an interplay between relativistic quantum mechanics and quantum optics. We gave two relevant examples: the *Zitterbewegung* and collapse-revival dynamics. In addition, we showed that the implementation of a 2D Dirac oscillator in a single trapped ion, with all analogies and measured observables, is at reach with current technology.

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- [1] D. Leibfried, R. Blatt, C. Monroe, D. Wineland, Rev. Mod. Phys. **75**, 281 (2003).
 - [2] D. Ito, K. Mori, E. Carrieri, N. Cimento **51 A**, 1119, (1967).
 - [3] M. Moshinsky, A. Szczepaniak, J. Phys. A **22**, L817, (1989).
 - [4] W. Greiner, “*Relativistic Quantum Mechanics: Wave Equations*”, (Springer, Berlin, 2000).
 - [5] L. J. Garay, J. R. Anglin, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **85**, 4643 (2000).
 - [6] P. M. Alsing, J. P. Dowling, and G. J. Milburn, Phys. Rev. Lett. **94**, 220401 (2005).
 - [7] J. Schliemann, D. Loss, and R. M. Westervelt, Phys. Rev. Lett. **94**, 206801 (2005).
 - [8] L. Lamata, J. León, T. Schätz, and E. Solano, quant-ph/0701208.
 - [9] E. T. Jaynes, F. W. Cummings, Proc. IEEE **51**, 89 (1963).
 - [10] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
 - [11] A carrier interaction consists on a resonant classical driving onto a two-level system, while the red-sideband and the blue-sideband correspond to a JC and anti-JC (AJC) interactions, respectively.